Statistical Physics

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1 Introduction

For this project the aim is to differentiate classical and quantum behavior with the goal of properly explaining how statistical physics is not to follow each particle individually but encase them in a group and see how each of them are distributed among the energy states using distribution functions and average properties. We will describe the classical approach denoted as Maxwell-Boltzmann Distribution, along with Bose-Einstein Distribution (Bosons) and Fermi-Dirac (Fermions) distribution which handle more quantum behaviors. Statistical Analysis can count the number of different arrangements of microscopic properties, as oppose to describing each individual particles within the system. An important idea is to differentiate between micro and macro states. Macro states are energy distributions that are observable and can be measured (like Temperature); while micro states are the different arrangements of microscopic values within a single macro state. In a real life system there are too many micro state configurations that are too complex to properly analyze each one, that is why we we use a distribution function describing how energy is shared among the system.

2 Theory and Analysis

The number of micro states associated with its macro states is refereed to its multiplicity, from the book it states that all micro states are equally possible. Chapter 10 offers a formula that describes how many states are available and how many of them are occupied

$$N = \int dN = \int_0^\infty N(E)\, dE = V \int_0^\infty g(E)\, f(E)\, dE$$

N represents the total number of particles between and N(E) represents the number per unit interval at Energy E. V is the volume (which has to be included for the number due to the density of states per unit volume), f(E) is the distribution function of energy and g(E) as the density of states. g(E) is important because when doing

$$g(E)dE$$

is the number of available per unit volume in that interval (E and E + dE).

2.1 The Maxwell-Boltzmann distribution

When considering the distribution of a classical system, in this system we assume the density is relatively low, which means the spacing between particles is large compared to their de Broglie wavelength. The overall system in a Maxwell-Boltzmann distribution is a classical system even with individual particles having some amount of quantized energy. This behavior works well in order to describe gases under ordinary pressure and temperature.

$$f_{\text{MB}}(E) = A^{-1} e^{-E/kT}$$

where k is the Boltzmann constant, T is temperature, and as the energy increases the occupation probability drops off exponentially. A^{-1} is the normalization constant to ensure the number of particles in the system is fixed. The book shows us a problem using the Maxwell-Boltzmann distribution to find the number of molecules having energy between E and E + dE, in a gas at temperature T, filling a container volume V with N as the number of molecules.

$$dN = N(E)\,dE = V\,g(E)\,f_{\mathrm{MB}}(E)\,dE = A^{-1}\frac{4\pi V(2s+1)\sqrt{2}(mc^2)^{3/2}}{(hc)^3}E^{1/2}e^{-E/kT}\,dE$$

The normalization constant ensures that the total number of particles is N by integrating over all the energies:

$$N = \int dN = \int_0^\infty N(E)\,dE = A^{-1}\frac{4\pi V(2s+1)\sqrt{2}(mc^2)^{3/2}}{(hc)^3}\int_0^\infty E^{1/2}e^{-E/kT}\,dE$$

$$N(E) = Vg(E)f_{\mathrm{MB}}(E) = \frac{2N}{\sqrt{\pi}(kT)^{3/2}}E^{1/2}e^{-E/kT}$$

and from here we can obtain the energy distribution for gas molecules, and this equitation is identical to the Maxwell-Boltzmann energy distribution given in the book as well. The final equation states that the most probable energy distribution is about $\frac{1}{2}kT$ with it gradually falling to 0 as energy increases, meaning that it is extremely rare to find a molecule with much more energy than kT. The Maxwell-Boltzmann equation describes particles in their absolute classical limit, meaning particles are still independent form one another while the probability of occupying a state is dependent on their energy state or $e^{-E/kT}$. As the temperature increases their is a border distribution with the peak moving to higher energy states while lower temperatures have the opposite effect.

2.2 Bose-Einstein distribution

$f_{BE}(E) = \frac{1}{e^{(E-\mu)/T-1}}$ AS opposed to electrons that have a spin of 1/s, Bosons have an integer spin that go from (0,1,2,..). Bosons have a tendency to accumulate in the ground state, this makes them fundamentally different from a classical interpretation. When the energy approaches a chemical potential, that is when most of the particles accumulate in the ground state. IN cases where the chemcial potential is 0, the distribution simplifies leading to Planck's radiation Law. Photons are a type of Bosons, and here we have the calculation of the number of photons in the range E and E + dE. Since the photons are continuously created and destroyed the number of particles is not constant:

$$dN = N(E)\,dE = Vg(E)f_{BE}(E)\,dE = V\frac{8\pi}{(hc)^3}E^2\frac{1}{e^{E/kT}-1}\,dE$$

after knowing this we move to the actual contribution to the energy density from photons:

$$u(E)\,dE = \frac{EN(E)\,dE}{V} = \frac{8\pi E^3}{(hc)^3}\frac{1}{e^{E/kT}-1}\,dE$$

From here we integrate over the total energy density over all the photon energy is, meaning add the contributions from all energy levels:

$$U = \int_0^\infty u(E)\,dE = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{E^3}{e^{E/kT}-1}\,dE = \frac{8\pi(kT)^4}{(hc)^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx$$

$$U = \frac{8\pi^5 k^4}{15(hc)^3}T^4$$

This in the end just means that the energy for photons is proportional to T^4 meaning it exhibits similar characteristics to blackbody radiation. Bosons end up suppressing the high energy states due to their nature, leading the total energy to be finite as oppose to tending towards an infinity in classical. There is an increase in number of available states vs a decrease in the probability of the particles occupying said states.

2.3 Fermi-Dirac distribution

$f_{FD}(E) = \frac{1}{e^{(E-\mu)/T}+1}$ The Fermi-Dirac distribution states that fermions can't accumulate in one state, they follow what is known as the Pauli exclusion principle which means that no more than 1 particle can occupy a single energy state. At low temperatures there are still high energy readings off theses electrons because they keep trying to accumulate in higher and higher energy states, and from that is where you see a spike. Since valence electrons in metal room relatively freely without any restrictions, they travel around the metal similar to how a gas travels through an enclosed space. This gas obeys the Fermi-Dirac distribution. By a similar process, the equation below is the distribution of electrons between E and E + dE.

$$dN = N(E)\,dE = Vg(E)f_{FD}(E)\,dE = V\frac{8\pi\sqrt{2m^3}}{h^3}E^{1/2}\frac{1}{e^{(E-E_F)/kT}+1}\,dE$$

When metal is heated up to room temperature from 0 kelvin to 300; only a small amount of electrons are actually affected by the change, meaning very few electron move up from filled states to empty states above the fermi energy. We can find the fermi energy by normalizing the equation above so it contains a set amount of electrons. The fermi-Dirac has the value of 1 while the Bose-Einstein has a value of -1. This means bosons tend to cluster up together in the energy levels while fermions occupy each individual energy level, while low energy states are clusters in bosons, there is a sharp boundary at the fermi energy meaning there can only be one kind of particle per states (+1).

$$N = \int dN = \int_0^\infty N(E)\,dE = \frac{8\pi V\sqrt{2m^3}}{h^3}\int_0^\infty \frac{E^{1/2}}{e^{(E-E_F)/kT}+1}\,dE$$

$$N = \frac{8\pi V\sqrt{2m^3}}{h^3}\int_0^{E_F} E^{1/2}\,dE = \frac{8\pi V\sqrt{2m^3}}{h^3}\frac{2}{3}E_F^{3/2}$$

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$E_m = \frac{1}{N}\int_0^\infty EN(E)\,dE$$

$$E_m = \frac{3}{5}E_F$$

The average energy at T=0 is actually 3/5 of the fermi energy, meaning that even in low temperatures electrons still have a lot of energy due to them occupying different energy levels, so the energy in ground state is different from the energy at the fermi energy boundary.

3 Real World Applications and Discoveries

3.1 Bose-Einstein Condesates

Bose-Einstein condensates are gases cooled down at extremely low temperatures. "Condensates" refers to the particles condensing in the ground state. Since at low temperatures bosons aren't in an excited state since all of them occupying the lowest energy state. The macroscopic number of atoms merge into a single quantum state when

$$T < T_c$$

. As opposed to classical physics where condensation means the gas particles are turned into liquid. "Condensation" refers to all the particles condensing in the lowest state of energy while still remaining in gas form. A majority fraction of those particles in the ground state behave like one quantum system. The fraction of the particle si given by this equation

$$\frac{n_0}{N} = 1 - \left(\frac{T}{T_c}\right)$$

These BEC can be used to model system in order to study superconductivity, superfluity and vortex matter. I stumbled across a paper named "Observation of Vortex lattices in Bose Einstein Condensates"; it refers to "the quantization of circulation effecting the behavior of microscopic quantum systems". Vortexes are like tiny whirlpools in quantum field, a condensate can be subjected to this perturbation by revolving laser beams around it. They used these vortexes were used to study surface waves in trapped BEC. I find this fact to be quite interesting, in the actual experiments themselves they used what they describe as a radio-frequency "shield", to prevent high energy atoms from heating the condensate, with their fraction energy being about 90 percent. Right after they circulated the condensates, (stirring the fluid like gas), they stopped and allowed the gas to expand making the patterns easier to see. The lattices formed into triangular shapes (16,32, and even 130 vortices), which is actually common in most quantum systems to observer this behavior (a similar one are type II superconductors). These lattices were created depending on how strongly and quickly the condensates could be rotated, helping these scientist to observe the behaviors of these condensates. They actually used sodium atoms, ad were able to figure out that these states were much more predictable and stable than once originally thought.

3.2 Fermi-Dirac Hardware

Since its already known that with Fermions, they can only occupy one energy state, meaning that there are only two possibilities for the conclusion of the states, either they occupy them or they don't. Fermions include electrons, protons and neutrons. Since they are particles with half integer spins they obey the Pauli exclusion principle, this is why fermions don't collapse into the ground state energy level. In metals the electrons can be described as gases since their fermi energies are a few Evs compared to the atmosphere at room temperature which is about 0.025 eVs. We can see this in the Fermi-Dirac distribution with the average energy at T=0 is actually:

$$E_m = \frac{3}{5}E_F$$

. This is why electrons near the fermi energy can exchange so easily while lower level electrons can't interact as easily. For computer chips this means everything since they communicate in 1s and 0s and actually for quantum computing it is even more important since they rely on controlling quantum states called qubits. These qubits can come from superconductivity and semiconductor spin. Superconductors can carry electrical current without meeting any resistance. Electron form pairs called Cooper pairs; these electrons pair up together helping the current flow with no resistance. The Josephson Junction is extremely important in quantum chip design with the Krane Physics book describing it as a thin insulating layer placed between the superconductors. Electron pairs can tunnel through this thin layer causing what is known as a super current.

$$I = I_0 \sin(\phi_1 - \phi_3)$$

Most quantum chips are built from circuit containing these Josephson junctions. Meaning instead of reading it as 1s and 0s (wether there are electrons in the well or not); these junctions can store information in quantum states meaning the can seamlessly fluctuates between 1s and 0s. Krane talks about something called a SQUID, which stands for Superconducting Quantum Interference device. Made from a superconducting loop with two insulating barriers. Electrons tunnel through these barriers with the current depending in the phase difference between the wave functions. This makes SQUIDs extremely sensitive to magnetic field detectors making them extremely useful to test and study situations at low magnetic fields (even fields produced by the synapsis in the brain that fire of current).

4 Conclusion and References

4.1 Conclusion

Thanks to statistical mechanics we are able to use these varies distributions to separate quantum and classical characteristics. We are able to describe how particles interact with various energy states given their composition and their environment. Classical has no restrictions in energy states, meaning that they have more predictable behaviors. Going to quantum behaviors now, these distributions have more probabilistic patterns, with sharps peaks, instead of a smooth and predictable. For the presentation, we presented various distributions and graphs that matched the behaviors of each section. This paper is able to tell us that particles aren't just affected by temperature, energy and environment, but by their very nature. The reason some particles behave the way they do is just because they do and that is how they operate, and i find that fascinating, you can go into all the explanations all the details you want, but some of the things that you learn in physics just are, and even with all the simulations and experimenting. As mentioned for the project we presented distributions that mirror how Bosons, fermions and classical particles act as temperature goes to 0. For the classical particles, the particles just stop moving as T goes to 0 and their energy also tends toward 0. For bosons particles accumulate in the lowest energy states, and if that energy state is 0, then the particles are not interacting, however the occupation of that ground state becomes very large. For fermions as T tends to 0, due to the Pauli exclusion principle, the energy that fermion have at 0 temperature are 3/5 of the fermi energy, meaning that fermion still have some energy at T 0. This was a very fulfilling project as simulations were shown, and we were able to explain each distribution with an example for each and how they act in the physical and theoretical worlds.

4.2 References

Krane, Kenneth S. *Modern Physics.* 4th ed., Wiley, 2012

Tong, David, *Statistical Physics.* University of Cambridge, 2011 (PDF from my Statistical Mechanics class)